ELSEVIER

Physica B 294–295 (2001) 736–739

PHYSICA B

www.elsevier.com/locate/physb

Detection of earth rotation with a diamagnetically levitating gyroscope

A.K. Geim*, H.A.M.S. ter Tisha

High Field Magnet Laboratory, University of Nijmegen, Toernooiveld 1, 6525 ED Nijmegen, The Netherlands

Abstract

Strong magnetic fields allow levitation of apparently nonmagnetic substances due to their weak but not negligible diamagnetic response of about 10^{-5}. Importantly, the diamagnetic force compensates gravity on the level of individual atoms and molecules and, therefore, can be used to mimic a continuous zero-gravity environment that, otherwise, is only achievable on board of a space station. Here we employ this earth-bound low gravity to demonstrate a simple mechanical gyroscope with sensitivity already comparable to that achieved by quantum and military gyroscopes. Our gyroscope can serve as a "shooting range" for the development of precision orbiting gyroscopes that have been a subject of intensive discussions regarding possible tests of general relativity.

Keywords: Levitation; Gyroscope; Diamagnetism

1. Introduction

Although Foucault's pendulum allowed accurate measurements of the earth rotation more than a century ago, detection of very slow absolute rotations continues to present a formidable scientific challenge, and the earth rotation ($\Omega_E = 7.29 \times 10^{-5}$ rad/s) remains the common reference for precision gyrometric techniques [1–5]. Interest in such techniques is stimulated by applications in navigation and geophysical studies, as well as possible tests of general relativity and general laws of gravity and inertia [1–7].

The accuracy of traditional, mechanical gyroscopes is limited by drifts caused by a remnant unbalance of the rotor and friction in its bearing, and it is practically impossible to suppress such drifts to a level below $10^{-3}\,\Omega_E$. A new generation of quantum gyroscopes based on the interference of light or matter promise to rival the mechanical gyroscopes [1] and have recently demonstrated the absolute accuracy between 0.1% and 1% of Ω_E [4,5]. In order to detect much slower absolute rotations ($< 10^{-5}\,\Omega_E$), there is no other known way but sending a mechanical gyroscope in space where gyroscope's drifts are significantly diminished due to reduced gravity [6,7].

In this communication, we want to point out that ground-based gyrometric techniques can be radically improved by employing diamagnetic levitation which effectively creates conditions of reduced gravity. Suspension of a spinning insulating ball in a strong magnetic-field gradient and in vacuum is essentially frictionless and the remnant mechanical torques are greatly suppressed due to compensation of gravity on a molecular scale. The

*Corresponding author. Tel.: +31-24-3653061; fax: +31-24-3652440.

E-mail address: geim@sci.kun.nl (A.K. Geim).

0921-4526/01/$ - see front matter
PII: S0921-4526(00)00753-5

principle and all the essential technical details required for diamagnetic levitation are extensively discussed in literature [8–13], and here we discuss only the features specific for a levitating gyroscope.

2. Experiment

To demonstrate the feasibility of the diamagnetically levitating gyrometer, we have measured the earth rotation by using a 2 cm diameter plastic ball (asphericity < 5 μm) suspended in a magnetic field of about 15 T. Fig. 1 shows our experimental setup. The gravity within the rotor ball was compensated to $\approx 10^{-2}g$, for the given field distribution in the magnet [9,10]. The ball was spun by airflow to about 20 Hz around the horizontal axis. We were able to increase the rotation speed further by focusing a laser beam on the ball's edge. The air could be pumped out but generally it was not required for observation of the earth rotation as it became visible to the naked eye after about 10 min. We used a Bitter magnet whose large electricity consumption, unfortunately, limited the observation time to about 2 h. Using a telescopic lens, we measured the rotor precession in the horizontal plane with an accuracy of $\approx 1°$. Fig. 2 shows the precession angle versus time as found in our observations. The expected and observed precessions agree within the detection error, and the best fit to the two curves yields the rotation speed of 15.07° per hour (7.31×10^{-5} rad/s). Further improvement of the gyroscope was not attempted.

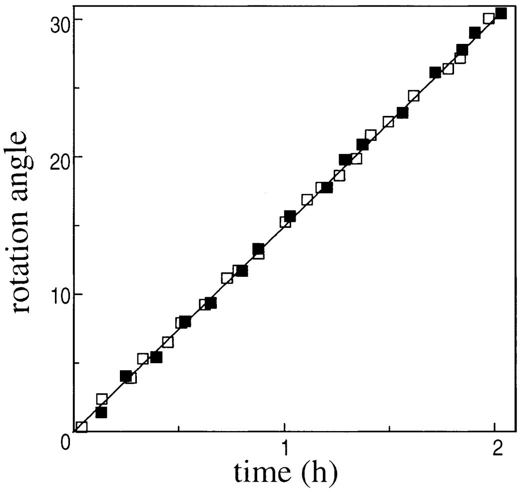

Fig. 2. Evolution of the spinning axis of our gyroscope for two series of measurements (symbols). After the initial spinning, we have waited for an hour for the precession to settle down before starting data collection. The accuracy of the optical detection was $\approx 1°$. The data were taken for the horizontal component of the precession and then normalized using the latitude of Nijmegen (51°50′). Alternatively, the data could be used to determine the latitude. Solid curve shows the expected rotation of 15° per hour.

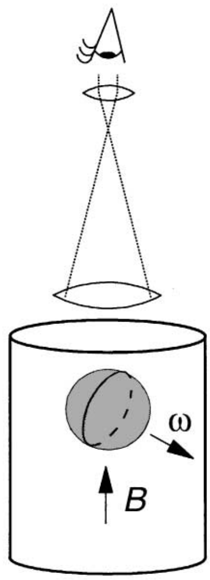

Fig. 1. Experimental setup. Plastic ball is spinning inside a Bitter magnet around the ω-axis. A pattern on the surface allows detection of changes in the ω-axis' orientation as earth rotates.

3. Drifts of a diamagnetically levitating gyroscope

Because the diamagnetic force counterbalances the force of gravity throughout the volume of a spinning ball, one can regard levitation as a simulated zero gravity. From this viewpoint, which is discussed in detail further in this section, the ultimate sensitivity of a diamagnetically levitating gyroscope will essentially be limited by accuracy with which the gravity is compensated, similar to the case of a space-based mechanical gyroscope.

We expect that major spurious drifts (beyond the registration error that is the limiting factor in our particular experimental setup) will be due to deviations of the field distribution from the perfect $B^2 \propto z$ dependence, which is required for the diamagnetic force to be constant along the vertical z-axis of the magnet [8–13]. In principle, a required magnetic-field profile can be created with a very high accuracy and 10^{-8} is routinely achieved in commercial superconducting systems. The second factor to be taken into account is that the presence of a diamagnetic rotor locally distorts the field profile. This distortion is of the order of diamagnetic susceptibility of the rotor, $\chi \approx 10^{-5}$, multiplied by a geometrical factor. The latter is zero for a perfect sphere and the field distortion by a nearly spherical, weakly magnetic ball is very small but not negligible (see below). Another important correction can appear because it is impossible to avoid magnetic forces in the direction perpendicular to the field axis [9,10]. This force arises due to the high-field gradient required for levitation and the simultaneous requirement of $\operatorname{div} B = 0$. We estimate that, in practice, the radial acceleration cannot be suppressed to the level better than $10^{-2}g$ in a few cm space. Fortunately, the radially symmetric acceleration does not cause precession as the relevant torques average out due to rapid spinning of the gyroscope.[1] Under the discussed circumstances, the only meaningful torque that causes spurious drifts, is due to asphericity of the gyroscope. The maximum torque T_m acting on a diamagnetically suspended ball of radius r and asphericity $\Delta r \ll r$ in a magnetic field B is

$$T_m = (12\pi/5\mu_0)(\mu - 1)^2/(\mu + 2)^2 B^2 r^2 \Delta r, \tag{1}$$

where $\mu = 1 + \chi$, i.e. $T_m \propto \chi^2$. Eq. (1) can be understood as follows. The presence of a weakly magnetic material distorts the magnetic field by a value of χB. This distortion produces an unbalanced magnetic moment $\chi.\chi B$ in the volume $r^2 \Delta r$ which interacts with the external field B and creates the detrimental torque T_m. The values of B and χ in Eq. (1) are interconnected via the equation [8–13]

$$mg = (4\pi r^3 \chi/3\mu_0)\partial B^2/\partial z, \tag{2}$$

which is required for the diamagnetic force to counterbalance the gravitational force mg. The diamagnetic force is linearly dependent on χ and, for a given magnetic configuration, we easily find that the detrimental torque is given by

$$T_m = \Gamma \chi m g \Delta r, \tag{3}$$

where $\Gamma \cong 1$ is a numerical factor that accounts for details of the gyroscope and field geometries.

Eq. (3) shows that spurious drifts linearly decrease with decreasing asphericity of a gyroscope and its susceptibility. A similar expression is also valid for an electrically levitating gyroscope. For example, in ground-based gyroscopes using AC electric fields for levitation [6,7], χ in Eq. (1) should be substituted by electric polarizability α. Since all solids have $\alpha \approx 1$, Eq. (1) clearly illustrates the major advantage of diamagnetic levitation with respect to electric (or superconducting) suspension: detrimental torques are suppressed by a factor of α/χ, i.e. a million times.

The above consideration agrees with our qualitative description of diamagnetic levitation as an effectively low-gravity environment. Furthermore, if the rotor material has a varying density ρ, such an inhomogeneity does not cause any additional torque, as the diamagnetic force depends on the ratio χ/ρ which remains constant for the same substance of varying density. Moreover, many diamagnetic substances have rather close values of χ/ρ so that minor inclusions of other diamagnetic materials can be expected to cause little torque.

Finally, we note that the diamagnetic levitation is inherently different from the well-known superconducting levitation which is often discussed in the context of precision gyrometers. For a superconducting suspension: (a) pinning usually leads to significant dissipation during rotation, (b) the supporting force acts only on the surface of a rotor, (c) $\chi = 1$ and (d) the induced London moment can cause a considerable additional torque.

[1] We cannot exclude that some corrections remain after such averaging. Dynamics of a gyroscope under a symmetric acceleration in cylindrical geometry is a complicated problem and requires further analysis.

4. Conclusion

We have shown that the detection of slow absolute rotations can be improved significantly by using the effective low-gravity conditions achieved in diamagnetic levitation. In our experiment, we have achieved the accuracy comparable to that of dedicated military and quantum gyroscopes. We believe that – combining a dedicated superconducting magnet with rapid spin ($>10^4$ Hz), precision manufacturing of the rotor (0.1 μm), optical read-out (10^{-11} rad), etc. [1–7] – it should be possible to build a diamagnetic gyroscope with an accuracy of several orders of magnitude better than that promised by other ground-based techniques. However, it remains to be shown in simulations and finally in an experiment whether our approach can provide the effective microgravity $<10^{-7}g$ required for tests of general relativity (drifts $<10^{-12}\,\Omega_E$) [6].

References

[1] C.W.F. Everitt, Nature 386 (1997) 551.
[2] A. Lenef et al., Phys. Rev. Lett. 78 (1997) 760.
[3] T.L. Gustavson, P. Bouyer, M.A. Kasevich, Phys. Rev. Lett. 78 (1997) 2046.
[4] O. Avenel, P. Hakonen, E. Varoquaux, Phys. Rev. Lett. 78 (1997) 3602.
[5] K. Schwab, N. Bruckner, R.E. Packard, Nature 386 (1997) 585.
[6] B. Lange, Phys. Rev. Lett. 74 (1994) 1904.
[7] B.G. Levi, Phys. Today 37 (1984) 20.
[8] E. Beaugnon, R. Tournier, J. Phys. III 1 (1991) 1423.
[9] M.V. Berry, A.K. Geim, Eur. J. Phys. 18 (1997) 307.
[10] M.D. Simon, A.K. Geim, J. Appl. Phys. 87 (2000) 6200.
[11] J.S. Brooks et al., J. Appl. Phys. 87 (2000) 6194.
[12] M. Motokawa et al., Physica B 256–258 (1998) 618.
[13] N. Kitamura et al., Jpn. J. Appl. Phys. 39 (2000) L324.