On optimal motorcycle braking

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Abstract

The optimal braking strategy in a high-performance motorbike is discussed. First, the control strategy using the front brake only is analyzed, highlighting the role of aerodynamics and studying how to select and modify the control objective during braking. The importance of the brake modulation in the very first part of the braking maneuver is also discussed. The role played by the rear brake is then analyzed. Finally, the attention is shifted to the damping ratio of the front suspension, which is probably the single motorbike parameter with the largest impact on vehicle dynamics during braking. A possible policy for semi-active suspension control during braking is sketched.

Keywords: Motorbike; Brake control; Semi-active suspension; Vehicle dynamics; Automotive systems

1. Introduction and motivation

In this work a simulation-based study of the braking maneuver in high-performance (racing) motorcycles is presented.

The starting point of this work is the analysis of a typical hard-braking maneuver performed by a professional driver. An example is given in Fig. 1, where the measured front-wheel speed, front-brake pressure and suspensions elongation are displayed (the measurements are made on the MY05 Aprilia RSV1000 Factory). During this braking maneuver, the driver makes no use of the rear brake. Notice that the brake pressure has a ramp (of about 250 ms), followed by a small brake release (probably due to an over-slip phenomenon sensed by the driver); then the brake pressure is kept almost constant until the end of the braking maneuver. By inspecting the elongation of the front and rear suspensions, it is interesting to observe that the suspension hard-limit is reached in both cases; notice that this means that the rear-wheel is at the contact limit; apparently, this may induce the driver to avoid turning the vehicle before the braking maneuver is ended.

Among pilots and race engineers it is common opinion that the braking phase is the most critical and sensitive maneuver. The ability of a driver to achieve an “optimal braking” can make the difference on the lap-time. Even few milliseconds per braking hence can be crucial. In this work, optimality simply means minimum time to decelerate the motorbike from the initial speed to a target speed. A broader range of performance parameters will be discussed in Section 4, where the sensitivity to the front-suspension damping will be analyzed.

The objective of this paper is to deeply analyze a single braking maneuver, trying to understand what the optimal maneuver should be, and the main parameters which can influence it.

The analysis is developed in the following setting:

- A pure braking maneuver is assumed, on a straight line (no lateral forces are engaged).
- An ideal driver is assumed; the concept of the ideal driver can be recast into an automatic closed-loop control system, following a reference signal (e.g., a target slip or a target load). The design of the reference signal is clearly a key issue for the optimality (“ideality”) of the driver.
The main challenge in the development of the “optimal maneuver” is due to the fact that a complete analytical model of a motorcycle is very complex, and it can hardly be used for the direct closed-form calculation of the optimal solution. On the other hand, reduced order models can indeed offer analytic solutions, but they cannot be really useful to find the optimum for the real target vehicle. This classical dilemma is solved according to the objective and the perspective of the specific research work; in this paper, since the objective is to analyze the very details of a braking maneuver, the simulator-based approach has been the natural choice (Fig. 2).

The analysis developed in this work is based on a full-fledged motorcycle simulator (the Mechanical Simulation Corp. BikeSim® simulation environment, based on the AutoSim symbolic multi-body software (Sharp, Evangelou, & Limebeer, 2004, 2005), which takes into account all the motorcycle dynamics, and accurately models the road–tire interaction forces (Sharp et al., 2004). See also Cossalter and Lot (2002), Donida, Ferretti, Savaresi, Schiavo, and Tanelli (2006), Frezza and Beghi (2005), Lin, Chyuan-Yow, and Tsai-Wen (2006), Sayers (1999), and Sharp and Limebeer (2001) for other state-of-the-art simulation environments suitable for this kind of analysis. The complete set of parameters used in the simulator is listed in Appendix A.

Solving the problem of the “optimal” maneuver is very attractive, since it allows to decouple the intrinsic performance of the vehicle from the driver behavior. In 2-wheel vehicles, these two aspects are so strictly inter-leaved that it is hard to predict the effect of a parameter change on the vehicle. This is a well-known problem, which sometimes has the picturesque effect of transforming the tuning of a racing motorbike from a rigorous model-based procedure into a sort of fine (or “magic”) art.
The optimal-maneuver problem is a very challenging task, which has been studied in the last decade by different research groups, from different perspectives (see e.g., Cossalter, Doria, & Lot, 1999; Cossalter, Lot, & Maggio, 2002; Frezza, Beghi, & Saccon, 2004; Hauser & Saccon, 2006; Hauser, Saccon, & Frezza, 2004; Limebeer, Sharp, & Evangelou, 2001; Sharp, 1971, 2001 for an overview of the literature on this topic). The specific problem of optimal braking has been considered in Cossalter, Lot, and Maggio (2004), where the focus was on steady-state conditions during a bend. Another paper dealing with the braking maneuver is Cossalter, Doria, and Lot (2000), where the design of the suspension for a scooter is discussed. The present work focuses on the dynamic aspects of a pure braking maneuver on a straight line.

In this paper the problem of the optimal braking maneuver is discussed following this path:

- Section 2. As first step, it is assumed that only the front brake is used (this is quite common, even in racing). Under this assumption the optimal control target is discussed; first this analysis is made by considering quasi-stationary conditions; then the fast transient of the very first part of the braking maneuver is analyzed.
- Section 3. The role and the benefits of rear braking are discussed.
- Section 4. Since the front-suspension damping is very critical for the overall braking performance, the influence of this parameter on the braking maneuver is analyzed.

The approach used in this paper differs from the classical automatic-control design path (system modeling, parameter identification, control algorithms design, performance evaluation). A mix of analysis of vehicle dynamics and closed-loop control architectures is proposed; as a matter of fact the primary goal of this paper is not the complete design of an automatic braking control system, but the understanding of the main dynamic phenomena and reference signals to be considered in the design of an optimal braking control strategy.

2. Optimal front-braking

In a high-performance motorbike it is customary to use only the front brake during a hard-braking maneuver. Since no front–rear coupling phenomena must be taken into account, and almost all the vehicle load is transferred onto the front wheel, the braking dynamics can be well-approximated with a single-corner model (Cossalter, 2002; Savaresi, Tinelli, & Cantoni, 2007; Solyom, Rantzler, & Lüdemann, 2004):

\[
\begin{align*}
J\dot{\omega} &= rF_x - T, \\
m\dot{\nu} &= -F_x,
\end{align*}
\]

where \(\omega\) is the angular speed of the wheel (it is expressed in rad/s; \(\omega > 0\) is assumed), \(v\) is the longitudinal speed of the vehicle body, \(T\) the braking torque which plays the role of control/input variable, \(F_x\) the longitudinal road–tire contact force, \(J, m\) and \(r\) are the rotational inertia of the wheel, the corner mass and the wheel radius, respectively.

The dynamic behavior of the system is hidden in the expression of \(F_x\), which depends on the state variables \(v\) and \(\omega\). The most general expression of \(F_x\) is quite complicated, since \(F_x\) depends on a large number of features of the road, tire and suspension; however, it can be well-approximated as \(F_x = F_{z\mu}(\lambda, \beta)\), where \(F_z\) is the vertical force at the tire–road contact point and \(\lambda\) is the longitudinal slip, given by \(\lambda = (\omega r - v)v = \mu_0 r - 1\) (the sign convention of negative slip when braking is used); \(\beta\) is the side-slip angle of the wheel. Since it is assumed that the braking maneuver is performed along a straight line \((\beta = 0)\), the dependence of \(\mu(\cdot)\) on \(\beta\) will be omitted.

The function \(\mu(\lambda)\) is called friction coefficient. It can be modeled using many empirical or semi-empirical parametric models, among which the most famous is probably the Pacejka “magic” formula (Pacejka, 2002; Savaresi, Tinelli, Langthaler, & Del Re, 2006). The curve \(\mu(\lambda)\) is characterized by a peak, which is typically around \(\lambda \approx -0.15\); this non-monotonic characteristic is responsible for the well-known fact that (for constant braking torque values) the equilibria associated to slip values beyond the peak of \(\mu(\lambda)\) are unstable. In Fig. 3, the road–wheel friction curve of the front tire used in the simulator is depicted, for different vertical contact forces \(F_z\). Notice that, at different vertical loads, this curve slightly changes. In the rest of the paper it is assumed that the maximum friction force is achieved at \(\lambda = -0.12\). This is rigorously true if \(F_z = 2000N\); since the total mass of the vehicle and driver is about 270 kg—see Appendix A—this vertical load corresponds to a significant load transfer onto the front axle. Notice that for higher vertical loads the peak position slightly changes, but by keeping the target slip at \(\lambda = -0.12\) the loss of performance is negligible (see details on Fig. 3).

The shape of the friction curve provides a seemingly simple solution to the optimal braking control target: in order to maximize the longitudinal force, the longitudinal slip \(\lambda\) should be simply regulated at the peak value of the friction curve, whatever is the vehicle speed and the vertical load.

Fig. 4 shows the results of a braking maneuver, using only the front brake and a target-slip control policy. The results displayed in Fig. 4 are obtained using a well-tuned PID slip regulator, applied to the Bikesim simulator (see Buckholz, 2002; Johansen, Petersen, Kalkkhuul, & Lüdemann, 2003 for details on the design of such a slip controller). The results displayed in Fig. 4 clearly show that—unfortunately—in a motorcycle the target-slip policy cannot be implemented, since the maximum braking torque can be higher than the overturn torque. This is a very peculiar feature of motorbikes (this problem does not exist in racing cars); it is clearly due to the high ratio between the height of the center of mass and the wheelbase (see Fig. 2).
By inspecting Fig. 4, it is worth pointing out that:

- the braking maneuver starts at a very high longitudinal speed (300 km/h);
- the PID-based automatic-control system is able to track very accurately (with both dynamic and static precision) the reference slip target $\tilde{\lambda} = -0.12$;
- the vertical load at the rear wheel ($F_{zr}$) rapidly decreases, while the pitch angle ($\theta$) increases; after 2.2 s from the braking start, at the speed of about 200 km/h and at the pitch angle of about $10^\circ$, the rear wheel starts losing contact with the road;
- the front-wheel target-slip is kept at $\tilde{\lambda} = -0.12$ even after the loss of contact; this causes an overturn of the vehicle (also called a “stoppie” in the motorcycle jargon—see photo in Fig. 4).

In order to better understand the reason of this phenomenon, a simplified model of the in-plane dynamics of the motorcycle can be used. Note that this model does
not consider the suspension dynamics; hence, it is valid for quasi-static conditions. The model is the following (subscripts “f” and “r” indicate front and rear parameters):

\[
\begin{align*}
  m\ddot{x} &= -F_x - C_D \dot{x}^2, \\
  m\ddot{z} &= F_{zr} + F_{zf} - mg + C_L \dot{x}^2, \\
  J_\theta \ddot{\theta} &= F_{zr}b - F_{zf}(p - b) + F_x h - C_P \dot{x}^2,
\end{align*}
\]

where

- \(p, b, h\) are the wheelbase, the distance between the projection of the center-of-mass on the road and the rear-wheel contact point, and the height of the center-of-mass, respectively (see Fig. 2);
- \(\{x, z, \theta\}\) are the longitudinal, vertical and pitch coordinates of the vehicle center-of-mass, respectively;
- \(m, J_\theta\) are the vehicle mass and its pitch inertia (around the center-of-mass), respectively;
- \(\{C_D, C_L, C_P\}\) are the drag, lift and pitch aerodynamic coefficients, respectively.

Using the simple model (2), two useful quantities, as functions of the forward speed \(\dot{x}\), can be derived.

The first is the deceleration \(\dot{x}_{\text{loss}}\) which causes the loss of contact of the rear wheel. It can be obtained by imposing \(\dot{\theta} = 0, \dot{z} = 0, F_{zf} = 0\). The result is the following:

\[
\dot{x}_{\text{loss}} = \frac{mg(p - b) - C_L(p - b)\dot{x}^2 + C_P \dot{x}^2}{m h} + C_D \dot{x}^2. \tag{3}
\]

The second is the deceleration \(\dot{x}_{\text{lock}}\) which causes the wheel lock-up; it can be obtained by imposing \(\dot{\theta} = 0, \dot{z} = 0, F_x = \mu_{\text{max}} \cdot F_{zf}\), where \(\mu_{\text{max}} = \mu(0.12)\). It is straightforward to show that when braking only with the front brake, this deceleration is related to the forward speed \(\dot{x}\) as follows:

\[
\dot{x}_{\text{lock}} = \frac{\mu_{\text{max}}}{(p - \mu_{\text{max}} h)m} (mg h - C_L \dot{x}^2 b - C_P \dot{x}^2) + C_D \dot{x}^2. \tag{4}
\]

The two variables (3)–(4) are plotted in Fig. 5. The parameter values used in (3) and (4) have been extracted by the Bikesim model, in order to obtain consistent results. Note that in (3) and (4) the sign of \(\dot{x}\) differs from (2), since the \(\dot{x}_{\text{loss}}\) and \(\dot{x}_{\text{lock}}\) are decelerations (they are positive when the vehicle speed decreases—see also Fig. 5). The results, although somehow intuitive, are interesting: at high speed, thanks to the large aerodynamic forces, the wheel-lock limit is dominant; this means that all the friction capability of the front wheel can be used, and the slip-target policy is optimal. When the speed decreases, so do the aerodynamic forces, and the rear-wheel contact limit becomes active (in the simulated vehicle this limit is about 200 km/h, see also Fig. 4). That speed is the “critical-speed” from the rear-wheel contact point of view. Below that speed value, in order to minimize the stopping distance while maintaining the vehicle lateral stability, the controlled variable must be switched from the front wheel slip to the rear vertical load. Also notice that, at low speed, \(\dot{x}_{\text{loss}}\) rapidly decreases, and the friction capability of the tires is significantly under-exploited.

The optimal braking strategy in a motorbike hence is twofold: at high speed a slip-control must be used, the reference signal being the peak of the friction curve; below the critical speed this control policy must be disengaged and replaced with a controller of the rear-wheel vertical load; in practice, since the critical speed is hard to estimate, the switching variable is the rear-wheel load. The control target of the load controller must be ideally \(F_{zr} = 0\); in practice, the zero-target has to be replaced with a small load (in the following, a load reference value of \(F_{zr} = 100\) has been used) to guarantee a minimum amount of lateral contact force on the rear wheel. This control architecture is pictorially described in Fig. 6. The results of this control policy are displayed in Fig. 7. The two SISO controllers of Fig. 6 are implemented with a standard PID architecture with anti-windup. Notice from Fig. 7 that the switch is seamless, since when the load-control PID is activated the state of its PI-part is not zero but it exactly equals the output of the slip controller at the switching time (this is implemented with the standard automatic-manual commutation algorithm for PID). In

![Fig. 5. Deceleration limits at different speeds.](image-url)
this way the switching guarantees a continuous behavior of the control variable.

From Fig. 7 notice that, when the rear-load controller is engaged, the front slip is progressively reduced, in order to avoid the loss of contact, while maximizing the braking performance. The braking maneuver of Fig. 7 can be considered optimal when only the front brake is used. Also notice that—as expected—the switch of the target occurs at a forward speed slightly higher than the critical speed, since $\lambda < 0$.

In the first part of this analysis the focus was on the general architecture of the control strategy. However, due to the complex dynamics of the vehicle and the tire, the very first part of the braking maneuver (say the first 200 ms) is particularly critical. This issue is now analyzed in detail.

The starting point of this analysis is the behavior of the automatic slip controller. This controller has been designed to provide the maximum achievable dynamic performance (maximum closed-loop bandwidth—see Savarese et al., 2007). The behavior of the control variable (front-braking torque) in the first 200 ms of the braking maneuver, when the target slip is set at $\lambda = -0.12$, is displayed in Fig. 8.

Notice that the controller initially requires a “pulse” of torque (about 10 ms long), due to the derivative part of the PID; then the torque is set to a small value and gradually increased again with a ramp-like behavior, until its steady-state value is reached. The ramp lasts about 150 ms. Notice,
however, that the control strategy implemented in Fig. 8 exceeds the actuator limit (saturation), and, in practice, the initial pulse is cut-off.

Starting from this ideal behavior, a sensitivity analysis on the effects of three different control strategies in this initial phase of the braking maneuver has been carried out. The three initial control strategies are (see the top sub-plot of Fig. 9):

- Control strategy 1. This control strategy simply mimics the behavior of the closed-loop slip controller: an initial short (10 ms) pulse, using the maximum available torque, is followed by a 140 ms ramp which starts from a value of about 40% of the final steady-state value.
- Control strategy 2. This control strategy mimics the behavior of a “real-driver” (see Fig. 1): the braking torque is increased from 0% to 100% of the steady-state value, with a 150 ms ramp (notice that a very fast-reacting driver has been assumed; in the measurement displayed in Fig. 1 the driver takes 250 ms to make the ramp).
- Control strategy 3. The idea of this control strategy is to immediately set the braking torque to the right

![Fig. 8. Initial behavior of the control variable (front braking torque) using a large-bandwidth slip controller.](image)

![Fig. 9. Sensitivity analysis to initial transient control strategy (from top: front braking torque; front-wheel slip; rear-wheel vertical load; vehicle speed).](image)
steady-state value; in practice this would cause wheel lock, since the load cannot be transferred to the front-wheel instantaneously, due to the suspension dynamics; hence, in practice, the braking torque is set to about 80% of the steady-state value, followed by a 150 ms ramp which closes the 20% gap.

The results of these three different control policies are condensed in Fig. 9, where the longitudinal slip, rear load, and forward speed of the vehicle are displayed. The following remarks are due:

- Even if the control strategies 1 and 3 show a remarkably different behavior in the longitudinal slip, their overall performances are very similar, in terms of vehicle deceleration. This can be explained with the fact that in the very initial part of the braking maneuver (20–30 ms) the load has not yet been fully transferred to the front wheel; thus, during this phase, the sensitivity to different values of wheel slip is comparatively small.
- The control strategy 2 shows significantly lower performance than 1 and 3. This is clearly due to the fact that the friction capability of the tire is under-exploited, since the magnitude of the wheel slip is increased too slowly.

It is interesting to observe that the control strategy 2 is the worst from the performance point of view, but it is the most natural and easiest for a "human" controller (see Fig. 1). On the other hand, control strategies 1 and 3 provide higher performance, but they can hardly be managed by a human driver. Note that the difference of performance in the 300→290 km/h deceleration is about 30 ms; it is non-negligible, considering that all this difference is condensed in the very first part of the braking maneuver. Hence, this implicitly confirms the potentially beneficial effect of an electronic brake assistant.

3. Optimal front–rear braking

In racing motorcycles the role and the importance of rear braking is very debated as, in practice, during a hard braking on a straight line, most of the drivers make no use at all of the rear brake. This brake is typically used for better managing the attitude and the stability of the motorbike during a bend. The reason is threefold:

- Due to the (almost) total load transfer to the front tire at mid-low speed in a hard-braking maneuver, the braking capability of the rear tire is comparatively small.
- The simultaneous optimal management of the front and rear brake is a very hard task even for a professional driver.
- If the engine is engaged during braking, the load torque of the engine (especially on high-performance high-compression 4-stroke engines) in many cases is high enough to lock the rear wheel; this phenomenon can be alleviated using mechanical devices called “anti-hopping” clutches, which can be considered as a raw anti-lock braking system acting on the engine-induced braking torque.

In order to simplify the analysis and to give a deep understanding of the role of the rear brake, it is assumed that the engine is disengaged during braking, and that all the rear-braking torque is provided by the rear brake.

As a starting point, it is interesting to analyze the behavior of the rear-wheel slip when the front-brake mixed slip-load control strategy illustrated in Figs. 6 and 7 (see the previous section) is used. The rear and front slip are displayed in Fig. 10.

From Fig. 10 it is interesting to point out that, at the rear wheel, the slip is positive, namely the rear wheel provides a traction torque, not a braking torque. This fact can be somewhat surprising at a first glance, but it can be easily explained by observing that the rear wheel has a kinetic energy given by $\frac{1}{2}J_r (\dot{\omega}_r)^2$, where $\dot{\omega}_r$ is the rear wheel rotational speed at the beginning of the braking maneuver, and $J_r$ the inertia of the rear wheel. If the engine is disengaged during braking and only the front brake is used, this energy is not dissipated but it is transformed into a traction torque.

When the rear brake is used, the control architecture of Fig. 6 must be changed into the control architecture of Fig. 11. The mixed slip-load strategy at the front brake is kept unchanged; the rear wheel is simply managed by regulating its slip $\lambda_r$ to the peak value of the rear friction curve.

The results obtained with this control architecture are displayed in Fig. 12. These results are compared with those obtained using the front brake only. The major difference is, obviously, in the slip of the rear wheel. As already noticed, this slip is positive (namely it provides traction torque) if no rear brake is used; if the rear brake is used, the rear slip is regulated to the same target ($\lambda_r = -0.12$) used for the front slip. Notice that the regulation of the rear slip is a difficult control task, due to the very small load on the rear wheel which causes a significant cross-disturbance of the front-brake controller onto the rear-slip dynamics. This non-perfect tracking of the rear wheel slip, however, has little effect on the overall performance.

This section is concluded by observing that in the long 300→80 km/h braking maneuver, the difference of performance is very large, about 300 ms. This difference is largely due to the fact that the “traction” torque has been replaced by a “braking” torque at the rear wheel. Although the braking torque at the rear wheel is rather small compared to the front-wheel one, on a strong braking maneuver the effect of the rear brake can be clearly appreciated. This is true, in particular, if the front-brake controller is able to maintain a small but non-zero load on the rear tire.

For the sake of completeness it is worth mentioning that, in practice, this performance improvement is significantly reduced if the engine is engaged and the vehicle is equipped...
Fig. 10. Front- and rear-wheel slip using a front-brake mixed slip-load control policy.

Fig. 11. Optimal control architecture of the front and rear brakes.

Fig. 12. Brake maneuver using the complete mixed slip-load control policy using the front and the rear brake (from top: longitudinal speed; front slip; rear slip; rear vertical load).
with anti-hopping clutch. From this point of view, the anti-hopping clutch can be interpreted as a very raw automatic rear slip controller. Obviously enough, a genuine electronic slip controller (acting on an electronically-controlled clutch or brake) can guarantee much better and more consistent slip-tracking performances.

This analysis demonstrates the potential benefits of an automatic control strategy for the rear brake: the braking performance is maximized while guaranteeing the road contact of the rear wheel. A somehow futuristic but very attractive way of achieving such high performance is the by-wire electro-mechanical brake (see e.g., Savaresi et al., 2007).

4. Sensitivity to front-suspension damping

In this section the effect of the damping ratio of the front suspension (fork) during a braking maneuver is discussed (see also Cossalter et al., 2000). As a matter of fact, among the many tunable parameters of a motorbike, the damping ratio of the fork is one of the most critical, since the vehicle dynamics are very sensitive to this parameter, and there is a strong trade-off in its tuning (see below). On this parameter, drivers and race engineers are always struggling in the search for the best compromise among different conflicting requirements.

The first part of the analysis has been developed in the frequency domain; more specifically, two transfer functions have been experimentally estimated using the simulator:

- the transfer function from the front braking torque \( T_f \) to the front wheel longitudinal slip \( \lambda_f \); this transfer function is useful in the design of slip controllers;
- the transfer function from the front braking torque \( T_f \) to the front wheel vertical contact force \( F_{zf} \); this transfer function is useful to investigate the minimization of vertical-load variations, which is a classical prerequisite to achieve high (and constant) longitudinal and lateral contact forces.

Since the braking maneuver is a non-equilibrium condition (due to vehicle deceleration), the computation of such transfer functions cannot be managed with traditional local-linearization tools, which typically require a well-defined steady-state condition; thus, an ad-hoc small-perturbation approach has been used (see e.g., Guardabassi & Savaresi, 2001; Savaresi & Spelta, 2007).

Fig. 13 shows a single step of this procedure: starting from a vehicle in straight running with a speed of 200 km/h, a constant front braking torque is applied; this constant torque generates (after a short transient) a steady-state front-wheel slip. Then a small sinusoidal perturbation is superimposed to the constant torque (a 2 Hz perturbation is depicted in Fig. 13); the corresponding slip is affected by a sinusoidal oscillation at the same frequency; by comparing the amplitude and phase relationships between the two sinusoidal oscillations, a single point of the estimated frequency response can be computed. This procedure has been repeated for a large number of frequencies (from 0.5 to 15 Hz, with a frequency resolution of 0.25 Hz).

The magnitudes of the estimated transfer functions are displayed in Fig. 14, for three different values of damping (low: 1000 Ns/m; medium: 3000 Ns/m; high: 5000 Ns/m). The following remarks can be made:

- In all cases, a strong trade-off is clearly visible: at low-damping all the transfer functions are characterized by a low-frequency (2.5 Hz) poorly-damped resonance; by increasing the damping ratio this resonance phenomenon first disappears (for medium damping) and then reappears as a poorly-damped resonance located at about 7 Hz.

![Fig. 13. Example of single-tone perturbation for frequency–response estimation.](image-url)
The resonance in the slip response is critical for the control of the wheel slip (both from a human-driver perspective and from an automatic-controller perspective). The resonance in the load-response to the braking torque is critical for the behavior of the contact forces.

In both cases, for low (below 5 Hz) and high (beyond 9 Hz) frequencies, the high-damping configuration is the best. The low-damping configuration outperforms it in the mid-range frequency range (5–9 Hz) only. This trade-off is well known for the vertical movement of the sprung and unsprung masses; it is interesting to see that this behavior also holds for the slip dynamics of the tire.

The frequency-domain interpretation has been complemented with a time-domain analysis. To this end, in Fig. 15 the time responses of vertical contact force ($F_z$), longitudinal contact force ($F_x$) and fork elongation ($\Delta x$), to a step in the front braking torque, are displayed for the three different levels of damping of the front suspension. All these responses refer to the front axle. By analyzing these step responses, additional insight in the influence of the damping ratio during braking can be gained. Some remarks are due.

- The first remark refers to the initial step response of the vertical contact force, in case of low damping. Notice a...
seemingly strange behavior: when a braking torque step is applied, the contact force initially decreases; then it increases and reaches its steady-state value. This behavior is not intuitive, since a load transfer from the rear to the front wheel is expected, whatever the damping ratio of the fork is. This unusual phenomenon is due to the fact that the angle with respect to the road surface of the front suspension of a motorbike significantly differs from $90^\circ$. The angle between the fork and a vertical line—called caster angle—for a sport motorbike is usually about $25^\circ$ ($26.1^\circ$ in our case). When a step on the braking torque is applied, a non-zero longitudinal braking force is rapidly generated; due to the non-zero caster angle, this force is not perpendicular to the front suspension; hence, the component of this force projected onto the fork axis tends to compress the suspension, unloading the tire (see Fig. 16). This phenomenon is particularly visible if the damping ratio is low. Obviously, this phenomenon is rapidly compensated by the load transfer from the rear to the front wheel. This remark shows that, at the very beginning of the braking maneuver, the best setting for the damping ratio is “high”.

From the stopping-time point of view, the most representative variable in a braking maneuver is the longitudinal contact force $F_{x}$, since it is the actual force which decelerates the vehicle. Hence, this force must be maximized at all times. By comparing the $F_{x}$ responses at low and high damping, it is interesting to observe that, in both cases, a resonance phenomenon occurs. In the case of low damping this phenomenon occurs at low frequency; in the case of high damping, at a higher frequency. This obviously confirms the frequency domain analysis. It is interesting to point out that the overall braking performance is indeed very similar with both damping values: a careful inspection of the two responses shows that the overall balance of the longitudinal force is approximately even, whatever the damping ratio value is. Notice that this substantial independence of the braking performance from the damping ratio no longer holds if a dynamic adjustment of the damping can be done (semi-active damping control—e.g., Savaresi, Bittanti, & Montiglio, 2005; Savaresi & Spelta, 2007). It is easy to see that a “high–medium–high” (or even “high–low–high”) damping switch during the first $300\text{ ms}$ of the braking maneuver would maximize the overall longitudinal forces. This brake-oriented semi-active control strategy (which is out of the scope of the present work) is currently under development.

Another important characteristic to be carefully considered during braking is the behavior of the fork compression, since this variable is directly “sensed” by the driver. By inspecting its step response for different damping ratios, it is clear that, with the low damping setting, the fork rapidly reaches its steady-state value. However, this fast transient is paid for overshoot and under-damped oscillations. On the other hand, the high damping setting guarantees no overshoot and no oscillations. In this case, the price to be paid is a slower response: the fork takes almost $500\text{ ms}$ to reach its steady-state condition. In a short braking maneuver followed by a bend, this slow transient is considered as a drawback, since the driver typically wants to “feel” a steady-state condition before engaging a turn. Also in this case, semi-active damping can help to alleviate the trade-off: notice that the “high–medium–high” switching policy above described can also speed up the transient of the high damping setting, while maintaining the attractive properties of no overshoot, no oscillations and no initial unload of the front wheel.

5. Conclusions and future work

In this work the problem of high-performance braking has been studied. The focus is on optimizing the braking performance, in terms of stopping time. All the main aspects of the problem have been considered and discussed: the effect of aerodynamics on the loss of contact of the rear wheel; the wheel-lock problem; the best torque modulation strategy at the beginning of the braking maneuver; and the role of the rear brake. Moreover, the sensitivity of the braking performance to fork damping has been studied.

The results of this analysis provide some useful insights on the problem of optimal braking. Moreover, this analysis shows the potential benefits of electronically assisted brakes: these benefits could be further exploited by joint design of braking control and semi-active suspension damping control.

Acknowledgments

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Appendix A. Motorcycle geometric and mechanical properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheelbase</td>
<td>1.370 m</td>
</tr>
<tr>
<td>Center of mass height (vehicle with rider)</td>
<td>0.621 m</td>
</tr>
<tr>
<td>Center of mass distance from front wheel</td>
<td>0.560 m</td>
</tr>
<tr>
<td>Caster angle</td>
<td>26.1°</td>
</tr>
<tr>
<td>Rear arm length</td>
<td>0.250 m</td>
</tr>
<tr>
<td>Mass of vehicle with rider</td>
<td>274.2 kg</td>
</tr>
<tr>
<td>Front wheel mass</td>
<td>12.7 kg</td>
</tr>
<tr>
<td>Rear wheel mass</td>
<td>14.7 kg</td>
</tr>
<tr>
<td>Frame pitch inertia</td>
<td>22 kg/m²</td>
</tr>
<tr>
<td>Front wheel moment of inertia</td>
<td>0.484 kg/m²</td>
</tr>
<tr>
<td>Rear wheel moment of inertia</td>
<td>0.638 kg/m²</td>
</tr>
<tr>
<td>Frontal cross section area</td>
<td>0.6 m²</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>0.52</td>
</tr>
<tr>
<td>Lift coefficient</td>
<td>0.085</td>
</tr>
<tr>
<td>Pitch coefficient</td>
<td>0.205</td>
</tr>
</tbody>
</table>

Rear suspension properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring stiffness</td>
<td>40,000 N/m</td>
</tr>
<tr>
<td>Spring pre-load</td>
<td>2080 N</td>
</tr>
<tr>
<td>Spring travel</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>10,000 Ns/m</td>
</tr>
<tr>
<td>End stroke stiffness</td>
<td>600,000 N/m</td>
</tr>
<tr>
<td>Swing arm mass</td>
<td>8 kg</td>
</tr>
</tbody>
</table>

Front suspension properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring stiffness</td>
<td>40,000 N/m</td>
</tr>
<tr>
<td>Spring pre-load</td>
<td>933 N</td>
</tr>
<tr>
<td>Spring travel</td>
<td>0.12 m</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>2500 Ns/m</td>
</tr>
<tr>
<td>End stroke stiffness</td>
<td>100,000 N/m</td>
</tr>
<tr>
<td>Front fork mass</td>
<td>7.25 kg</td>
</tr>
</tbody>
</table>

Tire properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial stiffness of front tire</td>
<td>130,000 N/m</td>
</tr>
<tr>
<td>Radial stiffness of rear tire</td>
<td>141,000 N/m</td>
</tr>
<tr>
<td>Paceika parameters of front tire</td>
<td>120/70, (Sharp, Evangelou, &amp; Limebeer, 2004)</td>
</tr>
<tr>
<td>Paceika parameters of rear tire</td>
<td>180/55, (Sharp, Evangelou, &amp; Limebeer, 2004)</td>
</tr>
</tbody>
</table>

References


In J. A. C. Ambrosio (Ed.), *Advances in computational multibody systems* (pp. 45–68). Dordrecht: Springer.
