Formability of a more randomly textured magnesium alloy sheet: Application of an improved warm sheet formability test

C.E. Dreyer \textsuperscript{a}, W.V. Chiu \textsuperscript{a}, R.H. Wagoner \textsuperscript{b}, S.R. Agnew \textsuperscript{a,*}

\textsuperscript{a} Department of Materials Science and Engineering, University of Virginia, 355 McCormick Rd, Charlottesville, VA 22904-4745, USA
\textsuperscript{b} Department of Materials Science and Engineering, The Ohio State University, 477 Watts Hall, 2041 College Rd, Columbus, OH 43210-1178, USA

1. Introduction

The forming limit diagram (FLD) provides a rather complete assessment of sheet metal formability under various strain paths. The concept of FLD was originally proposed by Keeler (1965), who concluded, from tests of various metals biaxially stretched over punches, that there was a critical ratio of major to minor strain that produced fracture. Goodwin (1968) combined Keeler’s data with fracture strains gathered on materials stretched under conditions of negative minor strain to produce the first rudimentary FLD. Currently, the standard approach to experimentally probe the FLD is to test strips of various widths, which enforce various strain paths, using the limiting dome height (LDH) apparatus (Hecker, 1975). The limit strains are generally measured post-mortem using grids printed or etched on the sheet surface prior to forming. It is almost universally observed that the minimum in the FLD is observed close to the plane strain tension condition (Wagoner et al., 1989), denoted FLD(0).

In fact, it has been shown that the majority of press-shop failures occur under conditions of plane strain tension (Wagoner et al., 1989). Traditional sheet formability tests, such as the Erickson/Olsen ball punch tests (ASTM E 643-84, 2000) primarily enforce biaxial stretching, rather than plane strain; hence, the results obtained often do not correlate well with practical formability. In addition, the results from these tests suffer from large uncertainties. The OSU formability test (OSUFT) was introduced by Miles (1991) and developed by Narasimhan et al. (1995) to enforce plane strain tension, while also simulating the typical die-sheet metal interaction. This test has been demonstrated to be highly reproducible in several studies, such as those conducted by Narasimhan et al. (1995) and Karthik et al. (2002), when compared to previously conceived plane strain formability testing procedures, including the use of the LDH apparatus (Ayers et al., 1984).

The objective of the present research is to determine the potential of the OSUFT for characterizing the formability of sheet materials at elevated temperatures, since a number of high specific strength materials exhibit poor room temperature formability, thus requiring warm forming. Post-mortem measurements of FLD(0) are compared with the data gathered during the test, in the form of load versus punch displacement curves. Establishing a relationship between the two would eliminate the need for time intensive measurement of strains of circle grids.

The formabilities of three magnesium alloys, including a recently developed alloy denoted ZW41, are assessed at temperatures in the range of $T = 160–350 \degree C$ using the OSUFT apparatus modified to test at elevated temperatures under isothermal conditions. Therefore, the results gathered can be used to determine the effect of temperature alone for validating constitutive models used in simulations. Notably, it has been claimed that alloy ZW41 has improved warm formability over traditional magnesium alloys...
Table 1
Nominal compositions (in wt%, balance Mg), grain sizes (μm), and texture strength (root mean squared value of the orientation distribution) of the three alloys.

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Al</th>
<th>Zn</th>
<th>Y</th>
<th>Zr</th>
<th>Grain size</th>
<th>Texture strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>AZ31</td>
<td>3</td>
<td>1</td>
<td>–</td>
<td>–</td>
<td>6.5</td>
<td>2.59</td>
</tr>
<tr>
<td>ZK10</td>
<td>–</td>
<td>1</td>
<td>–</td>
<td>0.3</td>
<td>20</td>
<td>2.03</td>
</tr>
<tr>
<td>ZW41</td>
<td>–</td>
<td>4</td>
<td>0.7</td>
<td>–</td>
<td>11</td>
<td>1.39</td>
</tr>
</tbody>
</table>

(Kim et al., 2006). A recent report of Chino et al. (2009) confirms that Mg–Zn–Y alloys can have exceptional stretch formability, even at room temperature. Explanations for the improved formability are sought in an attempt to develop insight required to inspire the design of new magnesium alloys with even further improved warm forming properties.

2. Experimental procedures

2.1. Materials and microstructure characterization

Formability testing was conducted on three magnesium alloys: AZ31, ZK10, and ZW41. The nominal compositions of the alloys are provided in Table 1. AZ31 and ZK10 are tradition magnesium alloys that were produced by DC casting, hot rolling, and warm-rolling to a final gage of 1.03 ± 0.02 mm. Alloy ZW41 was DC cast, extruded into a thin plate, and warm-rolled to a final gage of 1.02 ± 0.02 mm. All of the sheets were sheared into smaller 127 mm wide blanks. For the traditional alloys, the blanks were 127 mm in length. ZW41 tended to draw over the lock beads of the test apparatus so the blanks were cut with a length of 178 mm to increase the effect of the clamping force. Blanks were then electro-etched with a square grid of circles 0.254 mm in diameter for post-mortem strain measurements.

Metallographic samples were prepared by grinding with SiC abrasive paper and then polishing with 3 and 1 μm diamond suspensions. The polished samples were chemically etched with acetic picral (5 mL acetic acid, 6 g picric acid, 10 mL H2O, 100 mL ethanol). Each of the alloys exhibited roughly equiaxed grain structures typical of warm-worked Mg alloys that have undergone dynamic recrystallization during primary sheet processing (Fig. 1). The average lineal intercept grain sizes of the sheets are presented in Table 1.

Texture measurements were performed on samples ground to mid-plane using SiC abrasive paper, and then chemically etched in 20% Nital (20 mL HNO3, 80 mL methanol). A Scintag X1 diffractometer with Cu Kα sealed tube source (40 kV, 35 mA) was used to measure the basal (0002), prismatic (1010), and pyramidal (1011) pole figures on a 5° × 5° grid of tilt and azimuth. The resulting pole data were corrected for defocusing using experimental data from a randomly texture powder and analyzed using the Preferred Orientation Package-Los Alamos (popLA) software. Complete basal and prismatic pole figures are presented in Fig. 2, and the texture strengths (the root mean squared value of the orientation distribution function) are listed in Table 1. Alloy AZ31 has a strong basal texture, with the basal poles preferentially aligned with the sheet normal direction (ND). ZK10 has a similarly strong texture, but with basal poles of the peak texture component tilted slightly away from the ND toward the rolling direction (RD). ZW41 exhibits a much weaker texture which will figure prominently in the understanding of the improved formability that this alloy will be shown to possess.

2.2. Formability testing apparatus

The geometry of the OSUFT apparatus (Fig. 3) was based on that optimized by Narasimhan et al. (1995). The punch and die were machined from D2 tool steel (ASTM A 681-08, 2007), which is known for its wear and oxidation resistance at the moderate forming temperatures of interest. The binding plate was secured to the die by 8 (1/2 in.-20, grade 8 Zn-plated hex cap) bolts, as opposed to the hydraulic clamp normally implemented (Sriram and Wagoner, 1994). This allows the apparatus to entirely fit within a standard ATS Series 3620 convection furnace that, in turn, fits within standard computer controlled universal testing system (MTS Sintech 10/GL). Notably, there was no specific seasoning (Karthik et al., 2002) of the punch or binding plates, or lubrication of the sample, punch, or die. Rather, all contact surfaces were cleaned with acetone and polished daily.

The whole apparatus was heated to temperature prior to securing the blank. The furnace was then opened and the sample was
Tests were conducted at temperatures ranging from 150 to 350 °C and at a punch velocity of 1 mm/s. Analytical calculations (see Appendix) show that this correlates with an maximum Hosford (1979) effective strain rate of ~0.045 s\(^{-1}\). Because of the aforementioned texture and the likelihood of anisotropic behavior, tests were conducted both on blanks with the major straining direction aligned with the rolling (RD) and transverse (TD) directions. The punch load was recorded as a function of punch displacement, and it will be shown that the relative formability of the alloys is provided by these simple data alone. On-sample strain measurements were also made by comparing the length of the major and minor axes of etched circles immediately surrounding the fracture to the diameter of undeformed circles. Digital images of the fractured region (Fig. 4) were uploaded into Image Tool software (UTHSCSA, 2002), which was calibrated by measuring a line segment on the physical sample (with uncertainty of less than or equal to ±0.03 mm) using an optical microscope. Several methods of fully automatic image analysis were tested, and it was found that manually highlighting the ellipsoidal-grids on the digital image allowed data to be gathered accurately even when the etching was not of the highest quality or there was damage to the surface of the material.

### 3. Experimental results

On-sample strain measurements show that a large region of the sample indeed undergoes plane strain tension, including the location where failure initiates. Figs. 5(a)–(c) present the minor strain versus position for each of the alloys tested along the transverse direction (TD) at 275 °C. We observe a region in each where the minor strains are approximately zero, within a range of about ±0.05. There is a significant amount of minor strain close to the edges of the sample due to the lack of constraint, approaching uniaxial straining at the edge itself. We find that an average of 36 ± 3 mm or 28%, of the sample is under plane strain. Finally, using the technique illustrated by Ghosh et al. (1984), we determine the limit strains (the FLC) close to the plane strain condition on a forming limit diagram and extrapolate to obtain FLD(0) (see Fig. 5(d)–(f)).

#### 3.1. Punch load and displacement versus temperature

The punch load versus displacement curves from the three alloys collected at different temperatures and strain rates all show similar trends. There was a concave up region during initial contact (labeled (1) in Fig. 6(a)), followed by a nearly linear region labeled (2), and a departure from near linearity labeled (3). The samples tested at the lowest temperatures (T = 160–175 °C) failed abruptly within this non-linear region (3), and close inspection of these samples showed very little plastic localization prior to fracture. At higher temperatures, a fourth region (4) was observed in which the load reached a maximum, followed by failure (5). See Fig. 6 for an illustration of the trends in the load versus displacement curves at different temperatures. Two critical points in the load versus displacement curves were compared: the maximum load and the displacement at the point of fracture. Notably, all of the materials demonstrated a similar trend of monotonically decreasing punch load with increasing temperature (Fig. 7). The two curves (with the range denoting the scatter in the data points) superposed in Fig. 7 are scaled versions of the viscoplastic model of Sellars and Tegart (1967),

\[
\dot{\varepsilon} \exp \left( \frac{Q}{RT} \right) = A \cdot [\sinh(\eta \dot{\varepsilon})]^n,
\]

which, Neil and Agnew (2009) have shown well-describes the flow stress of Mg alloy, AZ31 over the warm forming temperature regime (T ~ 200–300 °C). This relationship will later be used...
Fig. 5. Minor strains in the free region of the sample (and in the vicinity of fracture) show a significant region of plane strain tension, and plotting major and minor strains in a forming limit diagram illustrates how the values of FLD(0) are obtained, (a and d) AZ31, (b and e) ZK10, and (c and f) ZW41.

In contrast with the simple trend observed for punch load versus temperature, the punch displacements at fracture show a rather complex trend for all three alloys (Fig. 8). Firstly, all the materials and test directions had local maxima at $\sim 215^\circ$C and local minima at 250°C, at temperatures above 250°C and below 215°C, the punch displacement at fracture increases monotonically with temperature. Notably, alloy ZW41 showed significantly higher punch displacements at all temperatures of 215°C and above. Taking an average value of 19 mm for the conventional alloys and a value of 26 mm for ZW41 (the average of the responses measured along RD and TD) suggests that ZW41 performs $\sim 35\%$ better than the conventional alloys at 215°C. None of the alloys exhibit strong anisotropy in the punch displacement at failure. Though the data is scant, it appears that alloy ZW41 may be anisotropic, since the RD samples show higher punch displacements at failure in comparison to TD.

3.2. FLD(0) versus temperature

The results of FLD(0) measurements re-emphasize that magnesium alloys exhibit acceptable formability at only mildly elevated temperatures. For example, the most common magnesium sheet alloy, AZ31, yielded a FLD(0) value of 0.32 at 215°C, which is comparable to automotive mild steel sheet tested at room temperature (Wagoner et al., 1989). Fig. 9 illustrates that FLD(0) values show the same complex trend with temperature that was observed for the punch displacement at fracture; in fact, the same “curves to guide the eye” that were developed for the punch displacement plot were scaled and superimposed on these data. Considering the FLD(0) val-
Fig. 6. Punch load vs. displacement as a function of temperature for three alloys, (a) AZ31, (b) ZK10, and (c) ZW41.

Fig. 7. Peak punch load vs. temperature for all three alloys tested along the rolling (RD) and transverse (TD). Curves provided to guide the eye are based upon an empirical relationship described in the text.

Fig. 8. Punch displacement at fracture vs. temperature for all three alloys tested along the rolling (RD) and transverse (TD) directions. Curves provided to guide the eye.

ues as a quantitative measure of formability, it is shown that alloy ZW41 performs \(~37\%\) better than the conventional alloy AZ31 at a temperature of 215 °C (FLD(0) \(\sim 0.44\) vs. \(\sim 0.32\)).

4. Discussion

4.1. Relative warm formability of Mg alloy sheets

In an attempt to explain the enhanced formability of ZW41, the constitutive behaviors of the alloys were compared. According to Agnew et al. (2008), the alloys examined in this study do not exhibit appreciable strain hardening within the warm forming temperature regime (\(T \sim 175–350\ °C\)). The strain rate sensitivities of the three alloys are also quite similar; at 200 °C, for example, all three alloys exhibited rate sensitivities of \(m = 0.12 \pm 0.02\), even though the formability of alloy ZW41 is most exceptional at that temperature (Agnew et al., 2008). Finally, the tensile flow stresses of the three alloys were similar to one another over the temperature regime of interest. In fact, the RD samples of alloy ZW41 were among the stronger samples tested in the study (Fig. 7). Thus, it does not appear that alloy ZW41 avoids a stress-based fracture criterion by simply flowing at a lower stress.

The grain size of the exceptional alloy, ZW41, is intermediate to the two conventional alloys, AZ31 and ZK10 (Table 1). On the other hand, the crystallographic texture of alloy ZW41 sheet is exceptionally weak, as compared to other Mg alloy sheets (Bohlen et al., 2007), including the AZ31 and ZK10 alloy sheets examined in this study (Fig. 2). This naturally calls attention to the plastic anisotropy of the alloys. Curiously, the room temperature strength anisotropy of alloy ZW41 is greater than the other two alloys (Agnew et al., 2008). This has to do with a detail of the texture of the alloys, namely
the non-trivial volume fraction of grains with basal poles nearly parallel to the TD in comparison to the very few grains oriented with basal poles parallel to the RD. This causes the soft mechanism of (1012) extension twinning to be active during TD tension, but not RD tension (Bohlen et al., 2007). This strength anisotropy is largely absent in the warm forming regime (Agnew et al., 2008).

The strain anisotropy of the three alloys was characterized by measurements of the r-values (Table 2) within the temperature range of interest. The tensile testing practice and procedure of r-value measurement applied to these alloys has been reported previously (Agnew et al., 2008). At room temperature, the two conventional alloys exhibit marked anisotropy with the transverse direction having an r-value much greater than 1, and the rolling direction having a lower r-value, $r_{RD} < r_{TD}$. On the other hand, the r-values of alloy ZW41 are lower than 1 at room temperature and a reversal in the sense of the in-plane anisotropy, $r_{TD} > r_{RD}$, is observed. The sense of the in-plane anisotropy remains constant for all three alloys at all temperatures examined; however, the level of in-plane anisotropy is significantly diminished for all three alloys within the temperature range, $T = 200–300 \ ^\circ C$, where significant formability is obtained. ZW41-TD samples have the lowest r-values and best formability, at all temperatures. However, this may be coincidental since a collective comparison of all the formability testing results (Figs. 8 and 9) with the r-value data in Table 2 does not reveal any other obvious trends. This latter observation is consistent with previous understanding that the forming limits of plane strain condition (FLD(0)), and on the stretching side of the formability plateau or decrease with temperature.

An explanation for the anomalous formability decrease with increasing temperature that occurred for each of the alloys at approximately 250 \ ^\circ C was sought. In a previous publication (Agnew et al., 2008), it was shown that an observed plateau in tensile elongation between 200 and 300 \ ^\circ C can be explained in terms of changes in constitutive behaviors, namely the strain hardening exponent and strain rate hardening exponent. However, this does not readily provide an explanation for the observed dramatic decrease in formability in-plane strain tension.

Dynamic strain aging (DSA) has been used to explain ductility decreases as the temperature is increased in a number of metal alloys, ranging from the well-known “blue brittleness” of steels to the poor room temperature formability of Al–Mg alloys. This is an attractive explanation since there have been a number of recent reports of DSA in rare earth-containing Mg alloys tested at

### Table 2

<table>
<thead>
<tr>
<th>Alloy</th>
<th>RT (Agnew et al., 2008)</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>AZ31</td>
<td>2.2, 4.0</td>
<td>0.93, 2.6</td>
<td>0.67, 1.7</td>
<td>0.70, 1.5</td>
<td>0.67, 1.2</td>
<td>0.88, 1.0</td>
</tr>
<tr>
<td>ZK10</td>
<td>0.69, 1.6</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>1.2, 1.4</td>
<td>0.83, 1.2</td>
</tr>
<tr>
<td>ZW41</td>
<td>0.95, 0.71</td>
<td>1.02, 0.80</td>
<td>0.94, 0.84</td>
<td>0.76, 0.66</td>
<td>0.66, 0.59</td>
<td>--</td>
</tr>
</tbody>
</table>

*The quantities in italics were measured on a sheet of the same alloy and H24 temper, but of different thickness, originally reported by Agnew and Duygulu (2005).*
Fig. 10. SEM fractographs of alloys (a and b) ZK10 and (c and d) ZW41 at 200 and 300 °C, respectively, show reduced propensity for catastrophic shear failure in the case of ZW41.

moderately elevated temperatures, Zhu and Nie (2004) observed serrated flow of a Mg–Y–Nd alloy tensile tested at temperatures between 150 and 215 °C, along with a yield strength plateau and some evidence of a negative strain rate sensitivity, all indications of DSA. Curiously, they noted the absence of a loss in ductility, which is often associated with DSA. Zhongjun et al. (2007) observed the same effects at temperatures between 200 and 300 °C in an Mg–Er alloy. Trojanová et al. (2005) present similar results for a number of rare earth alloys, including alloy ZE41 (where E denotes an Nd-rich rare earth mischmetal). Finally, Wang et al. (2007) present a historical overview of DSA observations in magnesium, along with a presentation centered on alloy ZE43 (where E denote Ce-rich mischmetal).

None of the characteristics of DSA, such as serrated flow, reduced strain rate sensitivity or yield strength anomaly, have been observed for the present ZW41 alloy sheet (Agnew et al., 2008). Perhaps a reason for the distinction is that all of the aforementioned studies of DSA in Mg–rare earth alloys appear to have been conducted on cast alloys, where yielding is known to be controlled by basal slip and/or tensile twinning. Bohlen et al. (2007) have shown that even weakly textured Mg alloy sheets, such as ZW41, still exhibit a significant non-basal slip activity at relatively low strains.

DSA has also been observed for non-rare earth-containing alloys, such as AZ91, close to room temperature (Corby et al., 2007); this observation is consistent with the expectation that the diffusivity of alloy elements like Al and Zn in Mg are higher than the diffusivities of rare earth elements. Curiously, alloys AZ31 and ZK10 examined in this study show a formability minimum at a similar elevated temperature to alloy ZW41 (Fig. 7). It is concluded that DSA may occur in magnesium alloys, but it does not appear to be responsible for the observed ductility minimum.

Most likely candidate explanations for the formability minimum are associated with the aforementioned transition in failure mechanism from one of localized shear instability (Fig. 10(a) and (c)) to necking (Fig. 10(b) and (d)). A connection with a transition in the dynamic recrystallization (DRX) mechanism, as observed by Galiyev et al. (2001), from localized shear band-related mechanisms at low temperatures to more uniform discontinuous dynamic recrystallization at the highest temperatures, is also likely. Jiang (2007) observed similar anomalous ductility trends in her uniaxial test data collected on extruded alloys, AZ31 and AM30, with a local minimum at 250 °C at a strain rate of 0.001 s⁻¹. In that study, incomplete DRX was illustrated to have led to local softening and premature failure; it is likely that the same phenomenon is responsible for the formability minimum observed in the present study, and none of the alloys examined are immune from the effect. Furthermore, Semiatin et al. (1998) have shown a relationship between the nucleation and growth kinetics of cavities and anomalous ductility trends in titanium alloy Ti–6Al–4V.

Careful metallographic examination of alloy AZ31 after testing in the OSUFT apparatus has revealed an expected trend with respect to DRX. A sample tested at 215 °C (Fig. 11a) shows evidence of DRX localized to the grain boundaries, a mechanism which has been associated with shear band formation in prior work of Ion et al. (1982). The AZ31 sample tested at 250 °C (Fig. 11b) has a microstructure that suggests more uniform DRX, yet there is still evidence that the material is susceptible to shear banding associated with cavity link up at that temperature. A note of caution is expressed regarding these micrographs; since the present implementation of the OSUFT apparatus prevents immediate quenching of the sample after deformation, it is very likely that some of the evidence provided for DRX is, instead, evidence of post-deformation static recrystallization.
As stated, the prior observations of a ductility anomaly in the vicinity of 250 °C by Jiang (2007) and Agnew et al. (2008) were at slightly lower strain rates than the present tests (1 mm/s punch speed correlates with a strain rate generally less than 0.05/s, Appendix). However, the temperature compensated strain rate (or Zener–Holloman parameter given by the left hand side of Eq. (1)) is much more strongly affected by temperature than by rate. Increasing from 200 to 250 °C will lower this parameter by a factor of greater than 25, given the typical activation energy of magnesium alloy hot deformation of about 135 kJ/mol, which is also an accepted value for the self-diffusion of Mg (Vagarali and Langdon, 1982). Even so, we do not suggest that the precise temperatures of the ductility transitions are the same for the various alloys and strain rates; rather, we observe that the basic trends are similar.

Future work should examine the effect of strain rate on formability, since Mg alloys have been shown to exhibit a monotonic increase in formability with decreasing strain rate at elevated temperatures (e.g., Abu-Farha and Khraisheh, 2007). Preliminary tests conducted at 275 °C at a higher punch velocity of 10 mm/s reveal another interesting aspect of the behavior of alloy ZW41. While the conventional alloys experienced a decrease in formability at the higher rate, ZW41 had a similar or even increased formability (Table 3).

### 4.3. OSUFT punch displacement as a measure of formability

The correlation between the punch displacement and FLD(0) datasets (Figs. 8 and 9), and the desire to develop a simple means of ranking sheet metal formability based upon the punch displacement data encouraged the development of a mathematical relationship between strain and punch stroke. We assume that the strain is uniform plane strain within the metal spanning between the die and punch, enabling a two-dimensional analysis based upon the schematic in Fig. 3. The free section of the half blank has an original length of \( x_0 \) when the punch displacement, \( d \), is zero. We examine two limiting cases to compute the strain: frictionless and complete sticking interactions between the sheet and the punch and die. The length, \( x \), of the half blank once the punch has traveled to a depth, \( d \), is the sum of the lengths of the arcs FG and BE and the segment EG (Fig. 3). It can be rewritten as

\[
x = x_0 \cos(\theta) - 2r \tan(\theta) + 2r\theta,
\]

since the arc lengths are both \( r\theta \), where \( r \) is the radius of the die and punch (\( r = 12.7 \) mm, in the present case). From similar triangles, we observe that the angle GHC and EGJ are equal to the angle \( \theta \). Recognize, further, that the triangle HCA is similar to GEJ. Therefore,

\[
\tan(\theta) = \frac{d - 2(r - r\cos(\theta))}{x_0 - 2r\sin(\theta)}.
\]
Solving for \( \theta \) with appropriate boundary conditions (e.g., \( \theta = 0 \) at \( d = 0 \); \( \theta \to \pi/2 \) as \( d \to \infty \)) gives:

\[
\theta(d) = -2 \tan^{-1}\left( \frac{x_0 - \sqrt{d^2 - 4rd + x_0^2}}{d - 4r} \right).
\] (4)

Under the assumption of no friction, the strain in the major direction is simply

\[
\varepsilon_{1,\text{no friction}}(d) = \int_{x_0}^{x(d)} \frac{dx'}{x} = \ln\left( \frac{x(d)}{x_0} \right) = \ln\left( \frac{x_0}{\cos(\theta)} - 2r\tan(\theta) + 2r\theta \right).
\] (5)

Under conditions of complete sticking, the portion of the sample that is involved in the straining, \( z \), is also evolving:

\[
z = \frac{x_0}{\cos(\theta)} - 2r\tan(\theta) = x - 2r\theta,
\] (6)

which leads to an expression similar to Eq. (5), though not with such a simple analytical solution:

\[
\varepsilon_{1,\text{friction}}(d) = \int_{x_0}^{x(d)} \frac{dx'}{z}.
\] (7)

Fig. 12 presents these two bounding solutions (Eqs. (5) and (7)) as a function of punch displacement. Superposed on the curve are the measured values of FLD(0) versus \( d \) at failure, irrespective of alloy, test direction, or test temperature. Quantitative comparison between the FLD(0) data and the analytical curves is not intended since the Eqs. (5) and (7) were developed under the assumption of uniform elongation, and the FLD(0) values include at least some post-uniform strain. However both the predicted curves and the experimental FLD(0) data demonstrate a monotonic relationship between strain and punch displacement, signifying that punch displacement is a valid metric to rank formability. It is worth noting that the FLD(0) values from alloy ZW41 appear to fall on a line of slightly lower slope than those from AZ31 and ZK10. This may be because, as was mentioned earlier, ZW41 had a slick surface finish in comparison to the other alloys, reducing the friction between the punch and die and the material. Further tests with lubricants would be necessary to determine if that is the case and how to reduce those effects.

4.4 Analysis of the punch load versus displacement

Calculations of the punch load versus displacement were performed to better understand the different regimes (1–5) highlighted in the experimental data (Fig. 6). Complete sticking (Eqs. (6) and (7)) was assumed, as well as steady state flow with no strain hardening, which is a reasonable assumption for these alloys in the temperature and strain regime of interest (Agnew et al., 2008).

As mentioned above, magnesium alloy sheets typically have strong crystallographic texture and attendant anisotropy. Thus, Hosford’s (1979) higher order anisotropic yield surface is used along with Eq. (1) to describe the major stress as

\[
\sigma_1(d) = \frac{1}{\eta} \left[ \frac{r_0(r + 1)}{r_0 + r_a^2 + r_0^2(1 - \alpha^2)} \right]^{1/\alpha} \times \sinh^{-1} \left( 2 A \dot{\varepsilon}_1(d) \left[ \frac{r_0(r + 1)}{r_0 + r_a^2 + r_0^2(1 - \alpha^2)} \right]^{1/\alpha} \dot{\varepsilon}^{0/RT} \right)^{\alpha}
\] (8)

where \( r \) and \( r_0 \) are the \( r \)-values in the direction of major and minor strain rates, \( \dot{\varepsilon}_1 \) and \( \dot{\varepsilon}_2 \), respectively; \( \alpha \) is the ratio between the major and minor stress, \( \sigma_1 \) and \( \sigma_2 \); and \( a \) is the higher order exponent used to alter the shape of the yield locus (see Appendix for a complete derivation). Combining the material constitutive rule, Eq. (8) with the geometry of Eq. (7), the punch load may be computed as

\[
P(d) = \sigma_1(d) |2 \sinh(\theta(d))| (w - t_0) e^{-\xi_1(d)},
\] (9)

where \( w \) is the sheet width and \( t_0 \) is the original thickness of the sheet.

Punch load versus displacement curves predicted at 275 \(^\circ\)C (Fig. 13) are plotted, again, using an activation energy \( Q = 135 \text{ kJ/mol} \), \( n = 6 \), \( \eta = 0.0107 \text{ MPa}^{-1} \), \( A = 2 \times 10^{11} \), similar to those used to fit tensile data by Neil and Agnew (2009). The shape of the modeled curves compare very well with the shape of the experimental curves (Fig. 6); the fact that the peak loads predicted at 275 \(^\circ\)C do not precisely match the experimental (they are under predicted) is most likely connected to the assumption of zero strain hardening. Notably, different \( a \) values do not have a gross impact upon this predicted flow stresses, which was expected since at 275 \(^\circ\)C, neither \( r_{BLD} \sim 0.78 \) nor \( r_{TD} \sim 1.1 \) are far from isotropy. Suitable selection of parameters in Eq. (8) allows degeneracy of Hosford’s (1979) yield locus to Hill’s (1950) quadratic, or von Mises isotropic yield locus.

The predictions also show that the maximum in the punch load versus displacement is not a signature of plastic instability, as it is in...
a constant strain rate tensile test. Rather, it appears to be the result of the simultaneous decrease in sheet thickness and a slowing in the increase in strain rate (Fig. 14). The precise displacement at which this peak in the load occurs will vary with the temperature, due to the varying dependence of the effective stress on the strain rate.

5. Conclusions

Magnesium alloy sheets do not typically exhibit useful formability at room temperature. In order to probe the formability at elevated temperatures, the OSU formability test was modified for isothermal testing up to 350 °C. It is shown that the test enforces plane strain over an appreciable portion of the sample, including the site of fracture initiation, and therefore can be used to determine FLD(0). Analysis shows that the major strain enforced on the sample can be simply correlated with punch displacement, which allows the relative formability of different sheet metals or forming conditions to be readily assessed. Punch load versus displacement behavior, including the observed peak in punch load, was qualitatively explained in terms of the test geometry and placement behavior, including the observed peak in punch load, forming conditions to be readily assessed. Punch load versus displacement, assuming complete sticking conditions.

Appendix. Derivation of major strain

In what follows, the major strain versus displacement at a given temperature is modeled using the Sellars–Tegart constitutive relation from Eq. (1). Using the chain rule, the strain rate of the material during the test is given by

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial \varepsilon}{\partial d} \frac{\partial d}{\partial t} = \frac{\partial \varepsilon}{\partial d} \cdot \nu,$$

where $\nu$ is the punch velocity, held constant during the test. It is shown that the strain rate within the sheet steadily increases during the test to a maximum and then decreases (Fig. 14). However, the maximum do not occur until a punch depth of approximately 30 mm, whereas all the samples in this study failed at values of 26 mm or less. For the critical range of punch displacement in which all the samples in this study failed (d = 15–30 mm), the strain rate increased a nominal amount from 0.032 to 0.058 s⁻¹ (Fig. 14).

Given a measure of 'effective stress' based upon a yield criterion, an associated 'effective strain' measure can be derived (Hosford and Caddell, 1993). Because strain anisotropy was measured, Hosford’s higher order anisotropic yield surface (Hosford, 1979) which was based on Hill’s model when $\alpha = 2$, though it has been shown that the r-value effects on flow stress are better captured by higher exponents (Hosford, 1979). In order to find the relationship between the minor and major strain, the enforced ratio of strain increments is used, again given by Hosford (1979) as

$$\rho = \frac{ds_2}{ds_1} = \left(\frac{r}{r_0}\right) \left(\frac{\alpha^{a-1} - r_0(1 - \alpha)^{a-1}}{1 + r_0(1 - \alpha)^{a-1}}\right),$$

where $\rho$ is the ratio of minor to major stress ($\sigma_2/\sigma_1$), and $d\sigma_1$ and $d\sigma_2$ are major and minor strain increments, respectively. Assuming plane strain ($d\sigma_2 = 0$), Eq. (12) can be solved for $\alpha$, yielding

$$\alpha = \frac{r_0^{1/\alpha-1}}{1 + r_0^{1/\alpha-1}}.$$  

Therefore, we can solve Eq. (11) for $\sigma_1$, giving

$$\sigma_1 = \sigma \left[ \frac{r_0(r + 1)}{r_0 + r_0^{(1 - \alpha)}} \right]^{1/\alpha}.$$  

The effective strain given by Hosford (1979) is

$$\bar{\varepsilon} = \varepsilon_1 \left(1 + \alpha \rho \right)^{\sigma_1/\bar{\sigma}},$$

For plane strain ($\rho = 0$), this combined with Eq. (14) gives

$$\bar{\varepsilon} = \varepsilon_1 \left[ \frac{r_0(r + 1)}{r_0 + r_0^{(1 - \alpha)}} \right]^{1/\alpha}.$$  

Combining Eqs. (14) and (16) with Eq. (1), and solving for the major stress, gives:

$$\sigma_1(d) = \frac{1}{\eta} \left[ \frac{r_0(r + 1)}{r_0 + r_0^{(1 - \alpha)}} \right]^{1/\alpha} \times \sinh^{-1} \left( \left( \frac{1}{\bar{\varepsilon}_1} \left[ \frac{r_0(r + 1)}{r_0 + r_0^{(1 - \alpha)}} \right]^{1/\alpha} e^{Q/RT} \right)^n \right).$$
References


